Commission Directive (EU) 2015/996 of 19 May 2015 establishing common noise assessment methods according to Directive 2002/49/EC of the European Parliament and of the Council (Text with EEA relevance)

ANNEX

Appendix E

The finite segment correction

This appendix outlines the derivation of the finite segment correction and the associated energy fraction algorithm described in Section 2.7.19.

E1 GEOMETRY

The energy fraction algorithm is based on the sound radiation of a 'fourth-power' 90-degree dipole sound source. This has directional characteristics which approximate those of jet aircraft sound, at least in the angular region that most influences sound event levels beneath and to the side of the aircraft flight path.
Figure E-1

Geometry between flight path and observer location O

Aircraft designation from ANP database	B727C3			
NPD-Identifier from ANP database	JT8E5			
No of engines	3			
Mode of operation	Departure			
Actual aircraft mass [t]	71,5			
Headwind [m/s]	5			
Temperature [°C]	15			
Airport elevation [m]	100			
Segment No	Mode	Target	Flaps	Engine Power
1	Takeoff		5	Takeoff
2	Initial Climb	Altitude 1 500 ft	5	Takeoff
3	Retract Flaps	210 kts IAS ROC 750 ft/min	0	Max. Climb
4	Accelerate	250 kts IAS ROC 1 500 ft/min	0	Max. Climb
5	Climb	10 000 ft	0	Max. Climb

Figure E-1 illustrates the geometry of sound propagation between the flight path and the observer location **O**. The aircraft at **P** is flying in still uniform air with a constant speed on a straight, level flight path. Its closest point of approach to the observer is $\mathbf{P_p}$. The parameters are:

d	distance from the observer to the aircraft
d_p	perpendicular distance from the observer to the flight path (slant distance)
q	distance from **P** to $\mathbf{P_p} = -V \cdot \tau$
V	speed of the aircraft

t	time at which the aircraft is at point **P**
t_p	time at which the aircraft is located at the point of closest approach $\mathbf{P_p}$
τ	flight time = time relative to time at $\mathbf{P_p}$ = $t - t_p$
ψ	angle between flight path and aircraft-observer vector

It should be noted that, since the flight time τ relative to the point of closest approach is negative when the aircraft is before the observer position (as shown in **Figure E-1**), the relative distance *q* to the point of closest approach becomes positive in that case. If the aircraft is ahead of the observer, *q* becomes negative.

E2 ESTIMATION OF THE ENERGY FRACTION

The basic concept of the energy fraction is to express the noise exposure *E* produced at the observer position from a flight path segment $\mathbf{P_1P_2}$ (with a start-point $\mathbf{P_1}$ and an end-point $\mathbf{P_2}$) by multiplying the exposure E_∞ from the whole infinite path flyby by a simple factor — the *energy fraction* factor *F*:

$E = F \cdot E_\infty$	(E-1)

Since the exposure can be expressed in terms of the time-integral of the mean-square (weighted) sound pressure level, i.e.

$E = const \times \int p^2(\tau)d\tau$	(E-2)

to calculate *E*, the mean-square pressure has to be expressed as a function of the known geometric and operational parameters. For a 90° dipole source,

$p^2 = p_p^2 \times \frac{d_p^2}{d^2} \times \sin^2\psi = p_p^2 \times \frac{d_p^4}{d^4}$	(E-3)

where p^2 and p_p^2 are the observed mean-square sound pressures produced by the aircraft as it passes points **P** and $\mathbf{P_p}$.

This relatively simple relationship has been found to provide a good simulation of jet aircraft noise, even though the real mechanisms involved are extremely complex. The term d_p^2/d^2 in equation E-3 describes just the mechanism of spherical spreading appropriate to a point source, an infinite sound speed and a uniform, non-dissipative atmosphere. All other physical effects — source directivity, finite sound speed, atmospheric absorption, Doppler-shift etc. — are implicitly covered by the $sin^2\psi$ term. This factor causes the mean square pressure to decrease inversely as d^4; whence the expression 'fourth power' source.

Introducing the substitutions

$$d^2 = d_p^2 + q^2 = d_p^2 + (V \times \tau)^2$$

and

$$\left(\frac{d}{d_p}\right)^2 = 1 + \left(\frac{V \times \tau}{d_p}\right)^2$$

the mean-square pressure can be expressed as a function of time (again disregarding sound propagation time):

$p^2 = p_P^2 \times \left(1+\left(\frac{V\times\tau}{d_p}\right)^2\right)^{-2}$	(E-4)

Putting this into equation (E-2) and performing the substitution

$\alpha = \frac{V\times\tau}{d_p}$	(E-5)

the sound exposure at the observer from the flypast between the time interval [τ_1,τ_2] can be expressed as

$E = const \times p_P^2 \times \frac{d_p}{V} \times \int_{\alpha_1}^{\alpha_2} \frac{1}{(1+\alpha^2)^2} d\alpha$	(E-6)

The solution of this integral is:

$E = const \times p_P^2 \times \frac{d_p}{V} \times \frac{1}{2}\left(\frac{\alpha_2}{1+\alpha_2^2} + \arctan\alpha_2 - \frac{\alpha_1}{1+\alpha_1^2} - \arctan\alpha_1\right)$	(E-7)

Integration over the interval [–∞,+∞] (i.e. over the whole infinite flight path) yields the following expression for the total exposure E_∞:

$E_\infty = const \times \frac{\pi}{2} \times p_P^2 \times \frac{d_p}{V}$	(E-8)

and hence the energy fraction according to equation E-1 is

$F = \frac{1}{\pi}\left(\frac{\alpha_2}{1+\alpha_2^2} + \arctan\alpha_2 - \frac{\alpha_1}{1+\alpha_1^2} - \arctan\alpha_1\right)$	(E-9)

E3 CONSISTENCY OF MAXIMUM AND TIME INTEGRATED METRICS — THE SCALED DISTANCE

A consequence of using the simple dipole model to define the energy fraction is that it implies a specific theoretical difference ΔL between the event noise levels L_{max} and L_E. If the contour model is to be internally consistent, this needs to equal the difference of the values determined from the NPD curves. A problem is that the NPD data are derived from actual aircraft noise measurements — which do not necessarily accord with the simple theory. The theory therefore needs an added element of flexibility. But in principal the variables α_1 and α_2 are determined by geometry and aircraft speed — thus leaving no further degrees of freedom. A solution is provided by the concept of a *scaled distance* d_λ as follows.

The exposure level $L_{E,\infty}$ as tabulated as a function of d_p in the ANP database for a reference speed V_{ref}, can be expressed as

$L_{E,\infty}(V_{ref}) = 10 \times \lg\left[\frac{\int_{-\infty}^{\infty} p^2 \times dt}{p_0^2 \times t_{ref}}\right]$	(E-10)

where p_0 is a standard reference pressure and t_{ref} is a reference time (= 1 s for SEL). For the actual speed V it becomes

$L_{E,\infty}(V) = L_{E,\infty}(V_{ref}) + 10 \times \lg\left(\frac{V_{ref}}{V}\right)$	(E-11)

Similarly the maximum event level L_{max} can be written

$L_{max} = 10 \times \lg\left[\frac{p_p^2}{p_0^2}\right]$	(E-12)

For the dipole source, using equations E-8, E-11 and E-12, noting that (from equations E-2 and E-8)
$\int_{-\infty}^{\infty} p^2 \times dt = \frac{\pi}{2} \times p_p^2 \times \frac{d_p}{V}$
, the difference ΔL can be written:

$\Delta L = L_{E,\infty} - L_{max} = 10 \times \lg\left[\frac{V}{V_{ref}} \times \left(\frac{\pi}{2} p_p^2 \frac{d_p}{V}\right) \times \frac{1}{p_0^2 \times t_{ref}}\right] - 10 \times \lg\left[\frac{p_p^2}{p_0^2}\right]$	(E-13)

This can only be equated to the value of ΔL determined from the NPD data if the slant distance d_p used to calculate the energy fraction is substituted by a *scaled distance* d_λ given by

$d_\lambda = \frac{2}{\pi} \times V_{ref} \times t_{ref} \times 10^{(L_{E,\infty} - L_{max})/10}$	(E-14a)

or

$d_\lambda = d_0 \times 10^{(L_{E,\infty} - L_{max})/10}$ with $d_0 = \frac{2}{\pi} \times V_{ref} \times t_{ref}$	(E-14b)

Replacing d_p by d_λ in equation E-5 and using the definition $q = V\tau$ from **Figure E-1** the parameters α_1 and α_2 in equation E-9 can be written (putting $q = q_1$ at the start-point and $q - \lambda = q_2$ at the endpoint of a flight path segment of length λ) as

$\alpha_1 = \frac{-q_1}{d_\lambda}$ and $\alpha_2 = \frac{-q_1 + \lambda}{d_\lambda}$	(E-15)

Having to replace the slant actual distance by scaled distance diminishes the simplicity of the fourth-power 90 degree dipole model. But as it is effectively calibrated *in situ* using data derived from measurements, the energy fraction algorithm can be regarded as semi-empirical rather than a pure theoretical.